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A New Mathematical Model for Traffic Systems: The Basic Traffic Unit

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ABSTRACT
In studying traffic congestions at toll plazas, a basic model for traffic systems was introduced with the hope to contribute towards a longer-term solution with the means for explaining and predicting congestions. The basic model is named the Basic Traffic Unit (BTU). Besides solving traffic congestion problems, the basic model may also be extended to other network problems. It is anticipated that future researchers may further this study using more specific operations research approach, based on simulation and queuing theory models which would then provide a better insight towards a more sustainable solution.

Keywords: Traffic Systems . Arrival Rate . Toll Plaza . Queueing Theory . Traffic Simulation

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1 INTRODUCTION

The road congestion problems not only happen in a big city but also prevalence in a small town. Many studies [1, 2, 3] have been conducted regarding road traffic problems and toll plazas were identified as one of the sources of traffic jams on expressways. In an attempt to understand traffic congestions in a proper way, we propose a model for traffic systems called Basic Traffic Unit (BTU) that can be combined into a traffic network, which can be used to explain observed/predict congestions. In studying toll plazas, some researchers adopt a hybrid method between queuing theory and simulation [4]. We introduced the possibility of using Basic Traffic Unit (BTU) and queuing theory with different arrival patterns for performance evaluation of the toll facility at International Conference on Information and Communication Technology in 2016. It is expected that the results may not be to the ultimate level aspired, but they can still be used as estimations for evaluating toll plaza operations. Detail explanation on the possibility of using BTU with queuing theory can be referred to our previous presentation [5].

2 BACKGROUND AND SOME CURRENT WORK

In general, a toll plaza consists of three components as given in Figure 1 – Queuing Area, Toll Booths, and Merging Area.

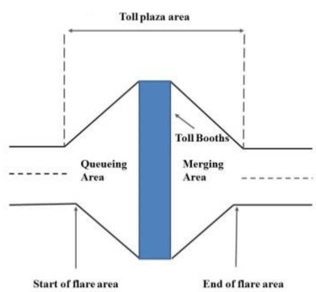

Fig. 1 Components of a toll plaza

Whenever heavy traffic approaches a toll plaza, temporary queues will build up before entering the booth area (a funnel effect at the Queuing Area), and there will be delays due to the time it takes to pay the toll (Toll Booths). Often, there would be more toll booths than the number of incoming lanes to distribute the queue and ease the traffic. However, the number of lanes after leaving the toll booths will have to get lesser to reform the original highway, and there will indeed be the possibility of another form of queue building up after paying the toll (a bottleneck at the Merging Area). The situation would be worse if the number of lanes after exiting the toll booths is much smaller than the number of incoming lanes; if there are additional feeder lanes from other highways before/after the toll plaza; if lane changing occurs due to different types of toll payment arrangements; and if there are junctions or traffic lights not far away from the toll plaza; etc. Solutions proposed for congestions at toll plazas will have to deal with at least three components, with considerations on the traffic flow and/or the toll plaza configurations: volume of arrival at the queuing area (queuing models); arrangement of the toll payment types of toll booths (lane changing and toll booth type models); and control of the volume of exit at the merging area (queuing models).

The approaches used would be based on the following (usually combinations of) in view of understanding the situations as well as to propose possible solutions: modeling the situations at the queuing area, toll booths, merging area; queuing theory and simulation; optimization. When randomly arising demands occur, there is a need to study the behavior of the system that attempts to provide services. In queueing theory, a model is constructed so that the six basics characteristics of queueing processes can be predicted and form an adequate description of the queueing system, namely: arrival patterns of vehicles, service patterns of servers (toll booths), queue discipline, system capacity, number of service channels – e.g. using cumulative curves [6], sensitivity analysis and environmental impact [7]. Other methods use computer simulation, discrete event simulation, continuous-time simulation [8, 9], or combinations of such approaches [10, 11, 12].

3 BASIC MODEL FOR TRAFFIC SYSTEMS

We propose a model for traffic systems in terms of basic traffic units that can be combined into a network, which can be used to explain observed/predict possible congestions. We can re-represent existing models in queuing theory and simulation into this representation, and take advantage of their formulae in a different light for further development. The complete interpretations can be found in Appendix 1 which is an adaptation from [5].

3.1 Example

The example of a combination of Basic Traffic Units to form a toll plaza can be found in Appendix 2 which is an adaptation from [5].

3.2 Method to Combine

Given the above, the method to combine should essentially be based on two base methods (see Figure 2) which are combining the results from two adjacent basic traffic units and combining results from *N* to 1 basic traffic units. At first glance, there may indeed be a need to look at more complex combinations, but further inspection may show that those Continuation points may and should be expressed as further basic traffic units – see the (yellow) circle in Figure 3 further down. We also consider queuing theory as a performance measure in future work. Figure 4 and Figure 5 clearly show the intended measurement to be used once appropriate data are collected. Figure 4 shows two BTUs in sequence and the appropriate measurements needed as well as the performance results by using queuing theory. Figure 5 demonstrates traffic network in sequence where it iteratively combined the result of the first two from the left, then combine the result with the next one, and so on until the last one. We shall not proceed any further on this as the computations may be very complex, hence left for future work.

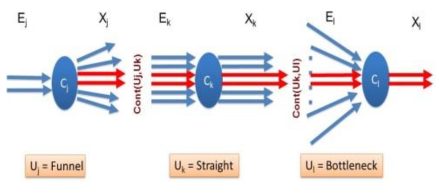

Fig. 2 Basis of method to combine

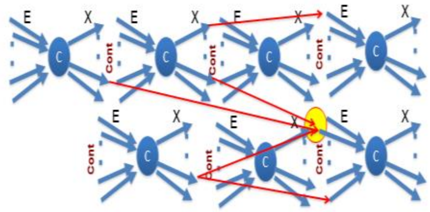

Fig. 3 Illustration of method to combine

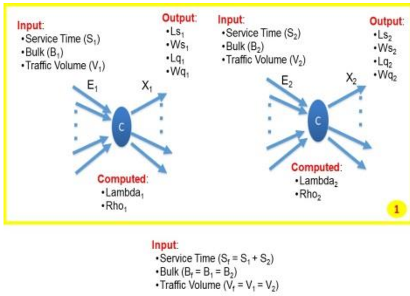

Fig. 4 Combination in sequence

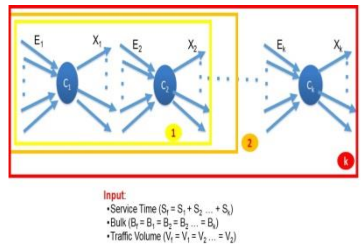

Fig. 5 Traffic network in sequence

4 CLEARANCE TIME

We proceed with estimation of the time it takes to clear a Basic Traffic Unit, and then a Traffic System, which will give an idea of the overall congestion time. The detail explanation of the generated formula on the measure of clearance time can be found in Appendix 3 which is an adaptation from [5].

5 IMMEDIATE FUTURE WORK

The main objective of the study is yet to be completed. An immediate approach can be as follows: (i) Divide a toll plaza into five zones, namely the lead-in area, the queuing area, the toll Affiliations. The booth, the merging area and the lead-out area (as indicated by the arrows in Figure 4). (ii) Based on work by [13], leading to a measure of performance, different arrival types can be considered and formulated. (iii) Develop a computer program to simulate traffic flow through the toll plaza. (iv) Analyze the data from this simulation and use it for planning more efficient traffic management. Further details can be found in [14].

6 CONCLUDING REMARKS

This paper has looked at traffic congestions at toll plazas in general, and at some current work. A basic model for traffic systems is introduced to be used as a long term solution to provide the means for explaining and predicting congestions. This can then be followed by a more specific operations research approach based on a simulation and queuing theory model which would then provide a better insight into a more sustainable solution.

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APPENDIX 1

Any traffic network can be decomposed into a network of Basic Traffic Units (and queueing theory may be applied to each unit, given the data). There should be a formula to combine results of queuing theory applications within a local network of Basic Traffic Units. These can be applied to formulate the necessary methodologies: for existing traffic networks – to explain and to predict/simulate traffic congestions; and for new traffic networks – to plan traffic networks to minimize future congestions. The principle is that problems are best solved when they are properly represented, and any formula used should be applied on basic components and then combined to form the final solution.

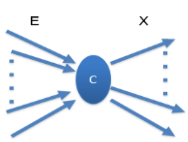

Fig. 6 A **Basic Traffic Unit** is a triple *(C, E, X)*

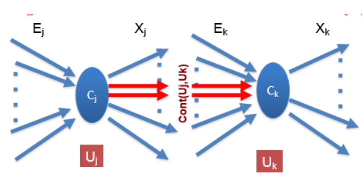

Fig. 7 A **Traffic System**

From Figure 6, C is the central point where all roads in the unit meet, which can be a junction flowing in one direction or a toll plaza, in which case (C, E, X) is a Toll Plaza Area. E is a set of roads entering C, with the number of roads being $|E|$, and X is a set of roads exiting C, with the number of roads being $|X|$. From the definitions above, C is a: **Bottleneck** if $|E| > |X|$; **Funnel** if $|E| < |X|$; **Straight** if $|E| = |X|$. From Figure 7, A **Traffic System** is a set of interconnecting Basic Traffic Units, i.e. a set of triples $U_i = (C_i, E_i, X_i)$ with $i = 1,2,..n$. $U_j = (C_j, E_j, X_j)$ is adjacent to and **precedes** $U_k = (C_k, E_{ik}, k_i)$ if Intersection (X_j, E_k) is not empty. If U_j precedes U_k, the meeting points of the exit roads X_j and the entry roads E_k will be referred to as the

continuing point $Cont(U_i\ ,\ U_k\ )$. $Cont(U_i\ ,\ U_k)$ can also be a bottleneck if $|\ X_j\ |\ >\ |\ E_k\ |$ (see the example below), which will which will be referred to as a **derived bottleneck.**

APPENDIX 2

Figure 8 displays a case of 4 adjacent basic traffic units U_i, U_j, U_k, U_l that form a full Toll Plaza area, where: $U_i = (C_i, E_i, X_i)$ is a straight leading into the Toll Plaza Area; $U_i = (C_j, E_j, X_j)$ is the main Toll Plaza area, with C_j being the toll booth, E_j the queuing area, and X_j the funnel or flare area, which precedes**;** a **straight** $U_k = (C_k, E_k, X_k)$, which is part of the merging area, followed by a **bottleneck** $U_l = (C_l, E_l, X_l)$

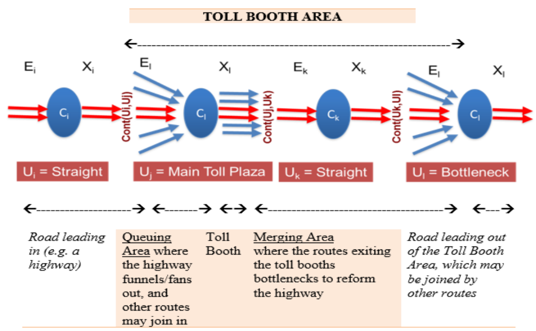

Fig. 8 Example of a Traffic System

The full merging area consists of X_j (the flare area) and U_k. The central points C_i, C_k and C_l are simply virtual points (not toll booths). The fourth basic traffic unit $U_l = (C_l, E_l, X_l)$ represents traffic moving out from the Toll Plaza Area, and possibly merging with other traffic sources. C_l is a point where all these traffic merge into a smaller number of road lanes. This situation can be regarded as the point where traffic from other sources merge into the same road after the Toll Plaza. Observe that: $Cont(U_i\ ,\ U_k)$ is a derived bottleneck because $|\ X_j\ |\ >\ |\ E_k\ |$; the congestion problem is caused by a sequence of two bottlenecks $Cont(U_i\ ,\ U_k\ )$ and C_i within a very short distance.

APPENDIX 3

Given a basic traffic unit (C,E,X). Let t be the time it takes for a vehicle to clear C, or the time it takes to clear a junction. We have the following; C is a single point – two toll booths would have to be two basic traffic units in parallel; the number of exit lanes $|\ X\ |$ is immaterial, as C can only process one vehicle at a time; the number of entry lanes $|\ E\ |$ is also quite immaterial, but rather the key is the number of cars arriving at C, say n, which would then congregate at the queuing area if $n \geq 2$ within the period t. When there are m (assumed equivalent) **toll booths** in parallel $[C_1, C_2, \ldots, C_m]$, making the usual toll plaza area. Then we have: Each C_i $(1 \leq i \leq m)$ clears a vehicle in time t, and as such m vehicles can be cleared within the same period t; The number of entry and exit lanes $|\ E_i\ |$ and $|\ X_i\ |$ $(1 \leq i \leq m)$ are still also quite immaterial for the same reason mentioned above; There will be a congregation at the collective queuing area if $n \geq m$ within the period t. When n cars arrive within the period t at the collective queuing area, the cars may choose any of the toll booths *m*, then we can assume that they will distribute evenly. For example, if 9 cars arrive at 3 toll booths, then they will distribute at 3+3+3, with 10 cars it would be 3+3+4, and so on. As such the clearance time for this example at a given toll booth will be either 3**t* or 4**t*. Given the above, the clearance time ***T*** at a toll plaza area for *n* cars arriving at *m* toll booths is (here the symbol ⌈ refer to 'rounding up to an integer value'):

$$\boldsymbol{T} = \lceil \mathbf{n/m} * \boldsymbol{t} \qquad (1)$$

For a **junction**, which is essentially a merging area where a larger number of lanes group into a smaller number of lanes, the case is simpler. There is no constraint on *C* to service one payment at a time, which is then equivalent to having the same number of 'booths' as there are exit lanes – in the case above $m = |E|$. The clearance time *J* for *n* cars arriving at a junction is thus:

$$\boldsymbol{J} = \lceil \mathbf{n}/|\boldsymbol{E}| * \boldsymbol{t} \qquad (2)$$

A toll plaza area or a junction may be loosely referred to as a bottleneck. It should follow from the above that if there is a sequence of *k* bottlenecks (see right hand side of Figure 5), the total clearance time for *n* cars arriving at the sequence would be the aggregate of the three following sums: the sum of each clearance time for all toll plaza areas in the sequence, the sum of each clearance time for all junctions in the sequence, the sum of the time taken to move from one bottleneck to another in the sequence (there are *k*-1 intervals). We note the following: The total clearance time is for the *n* cars arriving at C_1 within the period t_1 (the first unit in the sequence). Assuch, the value for *n* is the same for each C_i ($1 \le i \le k$). Each C_i ($1 \le i \le k$) will probably have a different t_i. Each C_i ($1 \le i \le k$) that is a toll plaza area will probably have a different m_i (the number of toll booths involved, Each C_i ($1 \le i \le k$) that is a junction will probably have a different $|X_i|$ (the number of exit lanes).

The following are more general notes on the overall performance of the sequence, which are consequences from possible changes in *m*, $|E|$, $\underline{t}$ and *m* in equations (1) and (2) above: From equation (1) above, it is clear that the clearance time at a toll plaza area will decrease with more toll booths *m* (as *T* decreases with a larger *m* as a denominator), But this advantage may be much reduced if the number of exit lanes $|E|$ in the junction that immediately follows is small, based on equation (2) above (as *J* increases with a smaller $|E|$ as a denominator).