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Abstracting Common Errors in the Learning of Time Intervals via Cognitive Diagnostic Assessment

Phei-Ling Tan, Liew-Kee Kor and Chap-Sam Lim

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ABSTRACT
This study engaged the cognitive diagnostic assessment (CDA) to abstract the common errors in the learning of time intervals based on pupils' knowledge states. CDA is a feasible testing tool that can inform us where a test taker may be prone to making errors in the tests. In this study, a cognitive diagnostic model with six attributes and 12 test items was created to evaluate pupils' performance in a diagnostic test on "duration of two inclusive dates". A total of 269 primary six pupils from 11 elementary schools participated in the study. The diagnostic test scores were analyzed using the Artificial Neural Network which generated 12 knowledge states (KS). Result shows that "100000" was the leading KS. The common errors associated with this KS in hierarchical order of prominence were: (i) exclude starting date as a day in duration; (ii) error in regrouping; (iii) compute incorrectly the sum of the two given dates; and, (iv) express incorrectly the time measurement in months and days. These identified common errors would provide a valuable basis for remedial teaching of the topic "Time". It also allows mathematics teachers to identify the inadequacy of an earlier teaching strategy and to engender an improved approach to help struggling learners shore up their basic skills.

Keywords: Cognitive diagnostic assessment. Knowledge state. Concepts of time. Time intervals. Primary mathematics

✉ Liew-Kee Kor
Universiti Teknologi MARA (UiTM) Kedah
E-mail: korlk7@gmail.com

Phei Ling Tan
SMK(P) Methodist, Penang, Malaysia
E-mail: tanpheiling@gmail.com

Chap Sam Lim
Universiti Sains Malaysia
E-mail: chpsam@gmail.com

1 INTRODUCTION

Good teaching allows teachers to teach higher level thinking, problem solving, and innovation. Despite the rigor in mathematics instructional studies, the scenarios "we teach but many students fear making mistakes" and "we assess but many students fear feedback and evaluation" still predominate most of the mathematics classroom [1].

Assessment indeed drives student learning and the characteristic of testing ideally is to provide meaningful and personalized feedback to a diversity of student learning and personal development [2]. According to Leighton, Chu and Seitz [1] cognitive diagnostic assessment (CDA) is a feasible testing tool that can inform us where a student may be prone to making errors in the tests. Furthermore, Borasi [3] pointed out that students' mathematical errors can be a powerful tool to diagnose learning difficulties and consequently direct remediation.
While CDA diagnoses errors, formative assessment provides us with a mechanism for remediating these errors. Adopting the definition that a learning error is a miscon-ception in the learning process that can be rectified [1] and using the schema theory, this study diagnosed how pupils learned the time concepts. Schema theory explains pupils' use their prior knowledge to solve current problems [4] while CDA diagnoses the stages of development of schematic knowledge within an individual [5].

This study investigated measurement of "time intervals" in the primary mathematics. "Time" is important in our daily life and yet it is complex and difficult for children to comprehend [6]. Closer inspection on TIMSS 2011 shows that 48% of the international Grade 4 pupils failed to answer correctly to a word problem on addition related to concepts of time [7]. Evidently learning about time can prove difficult for some students [8]. This study was conducted to evaluate pupils' mastery level of "time intervals" using the cognitive diagnostic assessment (CDA) tool (see details of CDA development in [9]); and, to conceptualize the learning errors from the analysis of pupils' knowledge states.

2 METHOD

2.1 Instrumentation

This study is part of a larger study which was conducted to develop a Cognitive Diagnostic Assessment (CDA) tool for the topic of Time in primary school learning [10]. The duration of two inclusive dates was identified the most difficult subtopic to mas-ter by most pupils in the topic "Time" [11]. The finished model consisted of six hierarchical attributes (Figure 1).

Attribute	Description of attribute	Hierarchy						Item
A0	Knowing number of days in a specific month	A0						1,2, 3
A1	Calculating duration that involves two inclusive dates within two consecutive months (for duration less than 30 days)	A0	A 1					4,5, 6
A2	Calculating duration that involves two inclusive dates within two consecutive months (for duration more than 30 days)	A0	A 1	A 2				7,8, 9
A3	Calculating duration that involves two inclusive dates more than two consecutive months but less than 5 months in duration.	A0	A 1	A 2	A 3			10, 11, 12
A4	Transforming word problem that involves two inclusive dates into mathematical operation	A0	A 1	A 2	A 3	A 4		10, 11, 12
A5	Expressing the final answer accurately as duration of days for word problems that involve two inclusive dates.	A0	A 1	A 2	A 3	A 4	A 5	10, 11, 12

Fig 1. List of attributes in duration of two inclusive dates

A total of 269 primary six pupils from 11 elementary schools participated in the study. Twelve items were developed to test the six attributes with three items to each attribute. Examples are items for attribute A0 (item 2): "List down all the months that have 30 days"; attribute A1 (item 4): "Find the duration from 15th September 2015 to 10th October 2015" and attribute A2 (item 7): "What was the duration from 11th May 2015 to 30th June 2015?" The reliability of the test instrument was 0.91 on the Cronbach alpha scale [11]. The test was administered to all primary six pupils from the 11 different primary schools.

3 RESULTS AND DISCUSSION

3.1 Pupils' Knowledge State

The cut-off value of the attribute probability (see Table 1and Table 2) of the test scores was calculated using the Artificial Neural Network (ANN). Based on the results, there were 12 knowledge states labelled from KS1 to KS 12 (Table 3). From the total of 269 respondents who answered the test, there were 78 respondents (29%) who could only master the first attribute of the cognitive model. This group of respondents could only answer (A0) "Knowing the number of days in a specific month" and were classified under knowledge state KS3 "100000" (Table 3).

Table 1. The cut off value of attribute probability and the knowledge state

Cut-off value of attribute probability (Calculated by ANN)	Classification of attribute probability	Value represented in knowledge state
<0.5	Non-mastery	0
0.5-0.8	Inconsistent-mastery	½
>0.8	Mastery	1

Table 2. Examples of six attribute probabilities and the knowledge state

Resp	Attribute Probability						Knowledge State
	1	2	3	4	5	6	
A	0.05	0.02	0.00	0.00	0.00	0.00	000000
B	1.00	1.00	0.99	0.02	0.00	0.00	111000
C	1.00	1.00	1.00	1.00	1.00	1.00	111111
D	1.00	1.00	1.00	0.95	0.80	0.00	1111½ 0

Table 3. The twelve extracted knowledge states

Label	Knowledge state	Frequency (%)	Cumulative %
KS1	000000	77 (28.62)	28.62
KS2	½ 00000	2 (0.74)	29.37
KS3	**100000**	**78 (29.00)**	**58.36**
KS4	1 ½ 0000	3 (1.12)	59.48
KS5	110000	34 (12.64)	72.12
KS6	11 ½ 000	3 (1.12)	73.23
KS7	111000	32 (11.90)	85.13
KS8	111 ½ 00	3 (1.12)	86.25
KS9	111100	4 (1.48)	87.73
KS10	111110	4 (1.49)	89.22
KS11	11111 ½	1 (0.37)	89.59
KS12	111111	28 (10.40)	100.00
	Total	269 (100)	

3.2 Types of Errors

Learning errors were detected from respondents' answers in the test items measuring attributes A0 to A5. These errors were finally grouped into four main categories. The categories with percentage occurrence are: (i) excluding starting date as a day in dura-tion (56.38%); (ii) error in regrouping (29.79%); (iii) expressing the sum of the two given dates as final answer (8.51%) and, (iv) expressing final answer in months and days (5.32%).

Excluding the starting date. This error secured the highest percentage (56.38%). Respondents made this error by excluding either the starting date or the ending date when counting the duration. The error of excluding the given dates occurred even when the calendar was provided. Pupils mistakenly thought that the starting date or the ending date should not be included in calculating the duration of the two given dates. The examples are shown in Figure 2 (with calendar) and Figure 3 (without calendar).

Fig 2. Error in excluding the starting date and ending date

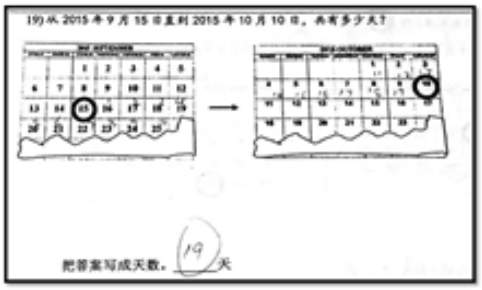

Fig 3. Error in excluding the starting date

As shown in Figure 2, a respondent wrote on the calendar that he/she was counting the duration in the two given dates. He/she did not include both the starting date and ending date in calculating the duration. As shown in Figure 3, there was no "+1" shown in the working steps which indicated that he/she did not include the starting date in calculating the duration. From this analysis, it could be inferred that majority of the pupils diagnosed with KS3 would probably make the error in excluding the given date as a day when calculating the duration.

Error in regrouping. This error has the second highest occurrence (29.79%). Even though KS3 inferred that pupils have mastered the basic attribute of knowing the number of days in a month, there were respondents who could not regroup a month to the correct number of days. In expressing the final answer in terms of days, errors were spotted at the last step of the solution, and during the process of subtraction. More than half of the occurrences of error of regrouping (64.29%) were at the last step of the solution, which is to regroup the months and days into the total number of days as final answer. The rest of the 35.71% of error of regrouping was due to respondents' mistake in regrouping the month into the incorrect number of days when they perform subtraction to find the duration of the two inclusive dates (Figure 4).

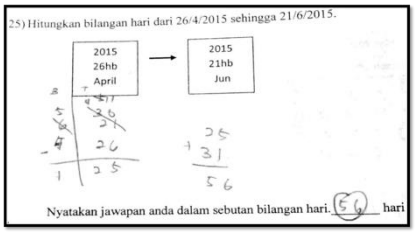

Fig 4. Error in regrouping the month (May) 30 days using subtraction

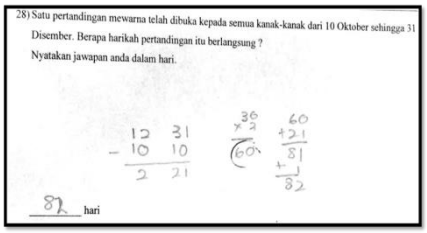

Fig 5. Example of regrouping a month into 30 days for all months

Of all the errors that occurred at the last step of regrouping, majority of the occur-rence (77.78%) was due to respondent regrouped a month into 30 days for all month or regrouped a month into 31 days for all the months (see Figure 4 and Figure 5). The other error was to regroup the months and days in the final step into the incorrect number of days (22.22%) with no significant pattern. This analysis inferred that for the pupils diagnosed with KS3, they might make the error of regrouping, particularly at the last step of the solution, in which pupils tended to regroup one month into 30 days for all month or regroup one month into 31 days for all month.

Expressing the sum of the two given dates as final answer (in months and days). The last two categories of errors were "expressed the sum of the two given dates as final answer" (8.51%) and "expressed final answer in months and days" (5.32%). The error of expressing the sum of the two given dates as final answer referring to pupils who added the two dates given and expressed it as the final answer. The error of expressing final answer in months and days was due to respondents mistakenly leaving the final answer in months and days instead of days only. This group of respondents did not regroup the months into the days and hence was given 0 score as they did not write the final answer in terms of days.

4 CONCLUSION

A cognitive diagnostic model describes the relationships between item performances and posited proficiencies [12]. This study sought to evaluate pupils' attained mastery level by engaging CDA assessment tool. Results show that most pupils (29%) were at the knowledge state of KS3 that is at level "100000". Furthermore, four common errors in solving duration problems were found at this level. They are (i) exclude starting date as a day in duration; (ii) error in re-grouping; (iii) compute incorrectly the sum of the two given dates; and, (iv) express incorrectly the time measurement in months and days. Most of the past studies investigating the difficulties in learning time concepts focused on teaching strategies [8]. The use of CDA to classified pupils' knowledge states in time concepts in this paper is unprecedented. The findings pro-vide a valuable basis for remedial teaching. Apparently the observed common errors will allow mathematics teachers to identify the inadequacy of an earlier teaching strategy and to engender an improved approach to help struggling learners shore up their basic skills in the topic "Time".

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Ethical approval

The data collected in this study was approved by the Educational Planning and Re-search Division (EPRD), Ministry of Education Malaysia, the Penang State Education Department, and the selected project schools in Penang, Malaysia.

Informed consent

Informed consent was obtained from all individual participants included in the study.

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